

UNIVERSITY PRESS, MSIDA, MALTA  
2018

MATHEMATICAL  
FORMULAE



THE UNIVERSITY OF MALTA  
DEPARTMENT OF MATHEMATICS



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**Table 4 (continued)**

(a)  $\alpha = 0.05$

$v_1$	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30
$v_2$															
<b>1</b>	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88	243.91	245.95	248.01	249.05	250.10
<b>2</b>	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45	19.46
<b>3</b>	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62
<b>4</b>	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75
<b>5</b>	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50
<b>6</b>	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81
<b>7</b>	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38
<b>8</b>	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08
<b>9</b>	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86
<b>10</b>	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70
<b>11</b>	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57
<b>12</b>	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47
<b>13</b>	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38
<b>14</b>	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31
<b>15</b>	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25
<b>16</b>	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19
<b>17</b>	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15
<b>18</b>	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11
<b>19</b>	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.43	2.38	2.31	2.23	2.16	2.11	2.07
<b>20</b>	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04
<b>21</b>	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01
<b>22</b>	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98
<b>23</b>	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96
<b>24</b>	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94
<b>25</b>	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92
<b>26</b>	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.90
<b>27</b>	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.93	1.88
<b>28</b>	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87
<b>29</b>	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.90	1.85
<b>30</b>	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84
<b>40</b>	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74
<b>60</b>	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65
<b>120</b>	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55
$\infty$	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46

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Table 4 (continued)

$v_1$	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	
$v_2$	1	647.79	799.50	864.16	899.58	921.85	937.11	948.22	956.66	963.28	968.63	976.71	984.87	993.10	997.25	1001.4
2	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.40	39.41	39.43	39.45	39.46	39.47	39.47	
3	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47	14.42	14.34	14.25	14.17	14.12	14.08	
4	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90	8.84	8.75	8.66	8.56	8.51	8.46	
5	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	6.62	6.52	6.43	6.33	6.28	6.23	
6	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52	5.46	5.37	5.27	5.17	5.12	5.07	
7	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82	4.76	4.67	4.57	4.47	4.36	4.30	
8	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36	4.30	4.20	4.10	4.00	3.95	3.89	
9	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03	3.96	3.87	3.77	3.67	3.61	3.56	
10	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78	3.72	3.62	3.52	3.42	3.37	3.31	
11	6.72	5.26	4.63	4.28	4.04	3.88	3.76	3.66	3.59	3.53	3.43	3.33	3.23	3.17	3.12	
12	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44	3.37	3.28	3.18	3.07	3.02	2.96	
13	6.41	4.97	4.35	4.00	3.77	3.60	3.48	3.39	3.31	3.25	3.15	3.05	2.95	2.89	2.84	
14	6.30	4.86	4.24	3.89	3.66	3.50	3.38	3.29	3.21	3.15	3.05	2.95	2.84	2.79	2.73	
15	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12	3.06	2.96	2.86	2.76	2.70	2.64	
16	6.12	4.69	4.08	3.73	3.50	3.34	3.22	3.12	3.05	2.99	2.89	2.79	2.68	2.63	2.57	
17	6.04	4.62	4.01	3.66	3.44	3.28	3.16	3.06	2.98	2.92	2.82	2.72	2.62	2.56	2.50	
18	5.98	4.56	3.95	3.60	3.38	3.22	3.10	3.01	2.93	2.87	2.77	2.67	2.56	2.50	2.45	
19	5.92	4.51	3.90	3.56	3.33	3.17	3.05	2.96	2.88	2.82	2.72	2.62	2.51	2.45	2.39	
20	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84	2.77	2.68	2.57	2.46	2.41	2.35	
21	5.83	4.42	3.82	3.48	3.25	3.09	2.97	2.87	2.80	2.73	2.64	2.53	2.42	2.37	2.31	
22	5.79	4.38	3.78	3.44	3.22	3.05	2.93	2.84	2.76	2.69	2.60	2.50	2.39	2.33	2.27	
23	5.75	4.35	3.75	3.41	3.18	3.02	2.90	2.81	2.73	2.67	2.57	2.47	2.36	2.30	2.24	
24	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70	2.64	2.54	2.44	2.33	2.27	2.21	
25	5.69	4.29	3.69	3.35	3.13	2.97	2.85	2.75	2.68	2.61	2.51	2.41	2.30	2.24	2.18	
26	5.66	4.27	3.67	3.33	3.10	2.94	2.82	2.73	2.65	2.59	2.49	2.39	2.28	2.22	2.16	
27	5.63	4.24	3.63	3.31	3.08	2.92	2.80	2.71	2.63	2.57	2.47	2.36	2.25	2.19	2.13	
28	5.61	4.22	3.63	3.29	3.06	2.90	2.78	2.69	2.61	2.55	2.45	2.34	2.23	2.17	2.11	
29	5.59	4.20	3.61	3.27	3.04	2.88	2.76	2.67	2.59	2.53	2.43	2.32	2.21	2.15	2.09	
30	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57	2.51	2.41	2.31	2.20	2.14	2.07	
40	5.42	4.05	3.46	3.13	2.90	2.74	2.62	2.53	2.45	2.39	2.29	2.18	2.07	2.01	1.94	
60	5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33	2.27	2.17	2.06	1.94	1.88	1.82	
120	5.15	3.80	3.23	2.89	2.67	2.52	2.39	2.30	2.22	2.16	2.05	1.94	1.82	1.76	1.69	
$\infty$	5.02	3.69	3.12	2.79	2.57	2.41	2.29	2.19	2.11	2.05	1.94	1.83	1.71	1.64	1.57	

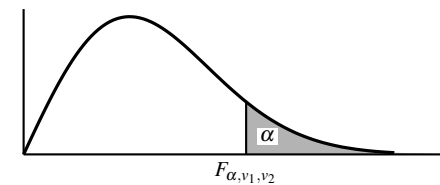
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**Table 4 Upper percentage points of the  $F$ -distribution**

(a)  $\alpha = 0.01$

$v_1$	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	
$v_2$																
<b>1</b>	4052.2	4999.5	5403.4	5624.6	5763.7	5859.0	5928.4	5981.1	6022.5	6055.8	6106.3	6157.3	6208.7	6234.6	6260.6	
<b>2</b>	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39	99.40	99.42	99.43	99.45	99.46	99.47	
<b>3</b>	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	27.23	27.05	26.87	26.69	26.60	26.51	
<b>4</b>	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55	14.37	14.20	14.02	13.93	13.84	
<b>5</b>	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05	9.89	9.72	9.55	9.47	9.38	
<b>6</b>	13.75	10.93	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.72	7.56	7.40	7.31	7.23	
<b>7</b>	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.47	6.31	6.16	6.07	5.99	
<b>8</b>	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.67	5.52	5.36	5.28	5.20	
<b>9</b>	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.11	4.96	4.81	4.73	4.65	
<b>10</b>	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.71	4.56	4.41	4.33	4.25	
<b>11</b>	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.40	4.25	4.10	4.02	3.94	
<b>12</b>	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.16	4.01	3.86	3.78	3.70	
<b>13</b>	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	3.96	3.82	3.67	3.59	3.51	
<b>14</b>	8.86	6.52	5.56	5.04	4.70	4.46	4.28	4.14	4.03	3.94	3.80	3.66	3.51	3.43	3.35	
<b>15</b>	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.90	3.81	3.67	3.52	3.37	3.29	3.21	
<b>16</b>	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.55	3.41	3.26	3.18	3.10	
<b>17</b>	8.40	6.11	5.19	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.46	3.31	3.16	3.08	3.00	
<b>18</b>	8.29	6.01	5.09	4.58	4.25	4.02	3.84	3.71	3.60	3.51	3.37	3.23	3.08	3.00	2.92	
<b>19</b>	8.19	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.30	3.15	3.00	2.93	2.84	
<b>20</b>	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.23	3.09	2.94	2.86	2.78	
<b>21</b>	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.17	3.03	2.88	2.80	2.72	
<b>22</b>	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.12	2.98	2.83	2.75	2.67	
<b>23</b>	7.88	5.66	4.77	4.26	3.94	3.71	3.54	3.41	3.30	3.21	3.07	2.93	2.78	2.70	2.62	
<b>24</b>	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.03	2.89	2.74	2.66	2.58	
<b>25</b>	7.77	5.57	4.68	4.18	3.86	3.63	3.46	3.32	3.22	3.13	2.99	2.85	2.70	2.62	2.54	
<b>26</b>	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	2.96	2.82	2.66	2.59	2.50	
<b>27</b>	7.68	5.49	4.60	4.11	3.79	3.56	3.39	3.26	3.15	3.06	2.93	2.78	2.63	2.55	2.47	
<b>28</b>	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.90	2.75	2.60	2.52	2.44	
<b>29</b>	7.60	5.42	4.54	4.05	3.73	3.50	3.33	3.20	3.09	3.01	2.87	2.73	2.57	2.50	2.41	
<b>30</b>	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.84	2.70	2.55	2.47	2.39	
<b>40</b>	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.67	2.52	2.37	2.29	2.20	
<b>60</b>	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.50	2.35	2.20	2.12	2.03	
<b>120</b>	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.34	2.19	2.04	1.95	1.86	
$\infty$	6.64	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.19	2.04	1.88	1.79	1.70	



# MENSURATION

Area of a circle, radius  $r$  is  $\pi r^2$

Circumference of circle is  $2\pi r$

*Sphere*

Volume of a sphere, radius  $r$ , is  $\frac{4}{3}\pi r^3$

Surface area of sphere is  $4\pi r^2$

*Right circular cylinder*

Volume of cylinder, radius  $r$  and height  $h$  is  $\pi r^2 h$

Curved surface area is  $2\pi r h$

*Right circular cone*

Volume of cone, radius  $r$ , and height  $h$  is  $\frac{1}{3}\pi r^2 h$

Curved surface area is  $\pi r l$  where  $l$  is the slant height of the cone.

$\alpha$	.005	.01	.025	.05	.10	.20	.50	.95	.975	.99	.995	100
1	0.000	0.0002	0.0010	0.0039	0.45	1.64	2.71	3.84	5.02	6.63	7.88	7.88
2	0.010	0.020	0.051	0.103	1.39	3.22	4.61	5.99	7.38	9.21	10.60	10.60
3	0.072	0.115	0.352	2.37	4.64	6.25	7.81	9.35	11.34	13.28	14.86	14.86
4	0.207	0.30	0.484	0.71	3.36	5.99	7.78	9.49	11.14	12.83	15.09	16.75
5	0.412	0.55	0.831	1.15	4.35	7.29	9.24	11.07	12.83	15.09	16.75	16.75
6	0.676	0.87	1.24	1.64	5.35	8.56	10.64	12.59	14.45	16.81	18.55	18.55
7	0.989	1.24	1.69	2.17	6.35	9.80	12.02	14.07	16.01	18.48	20.28	20.28
8	1.34	1.65	2.18	2.73	7.34	11.03	13.36	15.51	17.53	20.09	21.95	21.95
9	1.73	2.09	2.70	3.33	8.34	12.24	14.68	16.92	19.02	21.67	23.59	23.59
10	2.16	2.56	3.25	3.94	9.34	13.44	15.99	18.31	20.48	23.21	25.19	25.19
11	2.60	3.05	3.82	4.57	10.34	14.63	17.28	19.68	21.92	24.72	26.76	26.76
12	3.07	3.57	4.40	5.23	11.34	15.81	18.55	21.03	23.34	26.22	28.30	28.30
13	3.57	4.11	5.01	5.89	12.34	16.99	19.81	22.36	24.74	27.69	29.82	29.82
14	4.07	4.66	5.63	6.57	13.34	18.15	21.06	23.68	26.12	29.14	31.32	31.32
15	4.60	5.23	6.26	7.26	14.34	19.31	22.31	25.00	27.49	30.58	32.80	32.80
16	5.14	5.81	6.91	7.96	15.34	20.47	23.54	26.30	28.85	32.01	34.27	34.27
17	5.70	6.41	7.56	8.67	16.34	21.62	24.77	27.59	30.19	33.41	35.72	35.72
18	6.26	7.01	8.23	9.39	17.34	22.76	25.99	28.87	31.53	34.81	37.16	37.16
19	6.84	7.63	8.91	10.12	18.34	23.90	27.20	30.14	32.85	36.19	38.58	38.58
20	7.43	8.26	9.59	10.85	19.34	25.04	28.41	31.41	34.17	37.57	40.00	40.00
21	8.03	8.90	10.28	11.59	20.34	26.17	29.62	32.67	35.48	38.93	41.40	41.40
22	8.64	9.54	10.98	12.34	21.34	27.30	30.81	33.92	36.78	40.29	42.80	42.80
23	9.26	10.20	11.69	13.09	22.34	28.43	32.01	35.17	38.08	41.64	44.18	44.18
24	9.89	10.86	12.40	13.85	23.34	29.55	33.20	36.42	39.36	42.98	45.56	45.56
25	10.52	11.52	13.12	14.61	24.34	30.68	34.38	37.65	40.65	44.31	46.93	46.93
26	11.16	12.20	13.84	15.38	25.34	31.80	35.56	38.89	41.92	45.64	48.29	48.29
27	11.81	12.88	14.57	16.15	26.34	32.91	36.74	40.11	43.19	46.96	49.64	49.64
28	13.79	14.95	16.79	18.49	29.34	36.25	40.26	43.77	46.98	50.89	53.67	53.67
30	13.79	14.95	16.79	18.49	29.34	36.25	40.26	43.77	46.98	50.89	53.67	53.67
40	20.71	22.16	24.43	26.51	39.34	47.27	51.81	55.76	59.34	63.69	66.77	66.77
50	27.99	29.71	32.36	34.76	49.33	58.16	63.17	67.51	71.41	76.15	79.49	79.49
60	35.53	37.48	40.48	43.19	59.33	68.97	74.40	79.08	83.30	88.38	91.95	91.95
70	43.28	45.44	48.76	51.74	69.33	79.71	85.53	90.53	95.02	100.43	104.2	104.2
80	51.17	53.34	57.15	60.39	79.33	90.41	96.58	101.88	106.63	112.33	116.3	116.3
90	59.20	61.75	65.85	69.13	89.33	101.05	107.57	113.15	118.14	124.12	128.3	128.3
100	67.33	70.06	74.22	77.93	99.33	111.67	118.50	124.34	129.56	135.81	140.2	140.2

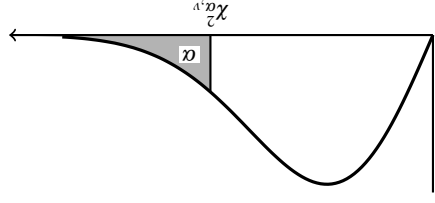


Table 3 Percentage points of the  $\chi^2$  distribution

## ALGEBRA

### Factors

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

### Permutations and Combinations

$${}^n C_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$${}^n P_r = \frac{n!}{(n-r)!}$$

### Finite Series

$$\sum_{q=0}^{n-1} (a + qd) = \frac{n}{2} [2a + (n-1)d]; \quad \sum_{q=0}^{n-1} ar^q = \frac{a(1-r^n)}{1-r}$$

$$\sum_{r=1}^n r = \frac{1}{2}n(n+1); \quad \sum_{r=1}^n r^2 = \frac{1}{6}n(n+1)(2n+1); \quad \sum_{r=1}^n r^3 = \frac{1}{4}n^2(n+1)^2$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1 \cdot 2} x^2 + \dots + \binom{n}{r} x^r + \dots + x^n \quad (n \text{ +ve int.})$$

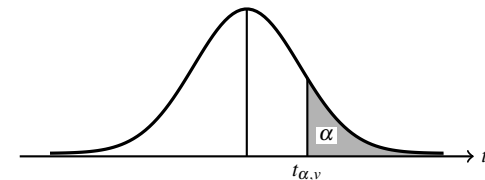
### de Moivre's Theorem

If  $n$  is an integer,  $(\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta$ .

If  $n$  is a rational number,  $\cos n\theta + i \sin n\theta$  is one of the values of  $(\cos \theta + i \sin \theta)^n$ .

**Table 2 Percentage points of Student's  $t$ -distribution**

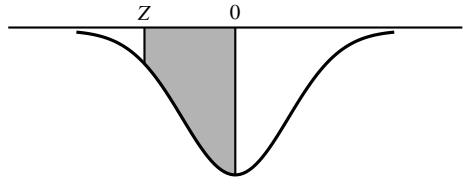
$\alpha$	.10	.05	.025	.01	.005	.001
$v$						
<b>1</b>	3.078	6.314	12.706	31.821	63.657	318.309
<b>2</b>	1.886	2.920	4.303	6.965	9.925	22.327
<b>3</b>	1.638	2.353	3.182	4.541	5.841	10.215
<b>4</b>	1.533	2.132	2.776	3.747	4.604	7.173
<b>5</b>	1.476	2.015	2.571	3.365	4.032	5.893
<b>6</b>	1.440	1.943	2.447	3.143	3.707	5.208
<b>7</b>	1.415	1.895	2.365	2.998	3.499	4.785
<b>8</b>	1.397	1.860	2.306	2.896	3.355	4.501
<b>9</b>	1.383	1.833	2.262	2.821	3.250	4.297
<b>10</b>	1.372	1.812	2.228	2.764	3.169	4.144
<b>11</b>	1.363	1.796	2.201	2.718	3.106	4.025
<b>12</b>	1.356	1.782	2.179	2.681	3.055	3.930
<b>13</b>	1.350	1.771	2.160	2.650	3.012	3.852
<b>14</b>	1.345	1.761	2.145	2.624	2.977	3.787
<b>15</b>	1.341	1.753	2.131	2.602	2.947	3.733
<b>16</b>	1.337	1.746	2.120	2.583	2.921	3.686
<b>17</b>	1.333	1.740	2.110	2.567	2.898	3.646
<b>18</b>	1.330	1.734	2.101	2.552	2.878	3.610
<b>19</b>	1.328	1.729	2.093	2.539	2.861	3.579
<b>20</b>	1.325	1.725	2.086	2.528	2.845	3.552
<b>21</b>	1.323	1.721	2.080	2.518	2.831	3.527
<b>22</b>	1.321	1.717	2.074	2.508	2.819	3.505
<b>23</b>	1.319	1.714	2.069	2.500	2.807	3.485
<b>24</b>	1.318	1.711	2.064	2.492	2.797	3.467
<b>25</b>	1.316	1.708	2.060	2.485	2.787	3.450
<b>26</b>	1.315	1.706	2.056	2.479	2.779	3.435
<b>27</b>	1.314	1.703	2.052	2.473	2.771	3.421
<b>28</b>	1.313	1.701	2.048	2.467	2.763	3.408
<b>29</b>	1.311	1.699	2.045	2.462	2.756	3.396
<b>30</b>	1.310	1.697	2.042	2.457	2.750	3.385
<b>40</b>	1.303	1.684	2.021	2.423	2.704	3.307
<b>60</b>	1.296	1.671	2.000	2.390	2.660	3.232
<b>120</b>	1.289	1.658	1.980	2.358	2.617	3.160
$\infty$	1.282	1.645	1.960	2.326	2.576	3.090





**Table 1 The standardised normal distribution**

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0754
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2258	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2518	.2549
0.7	.2580	.2612	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2996	.3023	.3051	.3079	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4430	.4441
1.6	.4452	.4463	.4474	.4485	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4700	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4762	.4767
2.0	.4773	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4865	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4949	.4951	.4952	.4954
2.6	.4955	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4978	.4979	.4980	.4981	.4981	.4981
2.9	.4981	.4982	.4983	.4983	.4984	.4984	.4984	.4985	.4985	.4986
3.0	.4986	.4986	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990
3.1	.4990	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4992
3.2	.4993	.4993	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995
3.3	.4995	.4995	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4998	.4998	.4998	.4998	.4999
3.5	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.6	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.7	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.8	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.9	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999



**HYPERBOLIC FUNCTIONS**

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\sinh^{-1} x = \ln[x + \sqrt{x^2 + 1}]$$

Principal value of  $\cosh^{-1} x = \ln[x + \sqrt{x^2 - 1}]$  ( $x \geq 1$ )

$$\tanh^{-1} x = \frac{1}{2} \ln \left| \frac{1+x}{1-x} \right| \quad (|x| < 1)$$

## CIRCULAR FUNCTIONS

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

$$\left. \begin{array}{l} \text{If } \sin \theta = \sin \alpha, \quad \text{then } \theta = n\pi + (-1)^n \alpha \\ \text{If } \cos \theta = \cos \alpha, \quad \text{then } \theta = 2n\pi \pm \alpha \\ \text{If } \tan \theta = \tan \alpha \quad \quad \text{then } \theta = n\pi + \alpha \end{array} \right\} \text{where } n = 0, \pm 1, \pm 2, \dots$$

$$\begin{aligned}\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\ \cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\ \tan(A \pm B) &= \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}\end{aligned}$$

$$\begin{aligned}\sin A + \sin B &= 2 \sin \frac{1}{2}(A+B) \cos \frac{1}{2}(A-B) \\ \sin A - \sin B &= 2 \cos \frac{1}{2}(A+B) \sin \frac{1}{2}(A-B) \\ \cos A + \cos B &= 2 \cos \frac{1}{2}(A+B) \cos \frac{1}{2}(A-B) \\ \cos A - \cos B &= -2 \sin \frac{1}{2}(A+B) \sin \frac{1}{2}(A-B)\end{aligned}$$

$$\begin{aligned}2 \sin A \cos B &= \sin(A+B) + \sin(A-B) \\ 2 \cos A \sin B &= \sin(A+B) - \sin(A-B) \\ 2 \cos A \cos B &= \cos(A+B) + \cos(A-B) \\ 2 \sin A \sin B &= \cos(A-B) - \cos(A+B)\end{aligned}$$

$$\begin{aligned}\sin 2A &= 2 \sin A \cos A \\ \cos 2A &= \cos^2 A - \sin^2 A = 1 - 2 \sin^2 A = 2 \cos^2 A - 1 \\ \tan 2A &= \frac{2 \tan A}{1 - \tan^2 A}\end{aligned}$$

$$\text{If } \tan \frac{A}{2} = t, \text{ then } \sin A = \frac{2t}{1+t^2}; \quad \cos A = \frac{1-t^2}{1+t^2}$$

*Paired sample t-test*

$$t_{n-1} = \frac{\bar{Y}}{S \sqrt{\left(\frac{1}{n}\right)}} \text{ where } Y_j = X_{1j} - X_{2j} \text{ (} j = 1, 2, 3, \dots, n \text{) and}$$

$$s^2 = \operatorname{Var}(y)$$

*Spearman's rank correlation coefficient  $\rho$*

$$\rho = 1 - \frac{6 \sum d^2}{n(n^2 - 1)}$$

*Kendall's rank correlation coefficient  $r$*

$$r = \frac{\left( \begin{array}{c} \text{Number of agreed} \\ \text{pair rankings} \end{array} \right) - \left( \begin{array}{c} \text{Number of different} \\ \text{pair rankings} \end{array} \right)}{\text{Number of pairs}} = \frac{S}{\frac{1}{2}n(n-1)}$$

*Paired sample Wilcoxon ( $n > 8$ )*

$T =$  (sum of the ranks with the less frequent sign)

$$Z = \frac{T - \bar{T}}{s} \text{ distributed } N(0, 1); \quad \bar{T} = \frac{n(n+1)}{4}; \quad s^2 = \frac{n(n+1)(2n+1)}{24}$$

## STATISTICS

$\mu, \sigma^2$  population mean and variance

$X_i$   $i$ th random selection in a sample size  $n$

*Sample mean*

$$\bar{X} = \frac{1}{n} \sum X_i$$

*Sample variance*

$$S^2 = \frac{1}{n-1} \sum (X_i - \bar{X})^2$$

$$E(S^2) = \sigma^2$$

$$E(\bar{X}) = \mu$$

$$\text{Var}(X) = \frac{\text{Var}(\bar{X})}{\sigma^2} = \frac{n}{n-1}$$

*One sample t-test*

$$\frac{\bar{X} - \mu_0}{S/\sqrt{n}} = t_{n-1}$$

*Two sample t-test*

$$t_{n_1+n_2-2} = \frac{\bar{X}_1 - \bar{X}_2}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$\text{where } S^2 = \frac{\sum (x_1 - \bar{x}_1)^2 + \sum (x_2 - \bar{x}_2)^2}{n_1 + n_2 - 2}$$

and  $n_1, n_2$  are the sizes of the two samples

## COORDINATE GEOMETRY

Perpendicular distance from  $(h, k)$  to  $ax + by + c = 0$  is  $\left| \frac{ah + bk + c}{\sqrt{a^2 + b^2}} \right|$

The acute angle between two lines with gradients  $m_1, m_2$  is

$$\tan^{-1} \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

**Area of Triangle is**

$$\frac{1}{2} \begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ 1 & 1 & 1 \end{vmatrix} = \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

**Circle**

The equation  $x^2 + y^2 + 2gx + 2fy + c = 0$  represents a circle with centre at  $(-g, -f)$  and radius  $\sqrt{g^2 + f^2 - c}$ .

The parametric equations of a circle with centre at  $(a, b)$  and radius  $r$  are

$$x = a + r \cos t, y = b + r \sin t.$$

Point dividing  $P_1 P_2$  in the ratio  $k : 1$  has coordinates

$$\left( \frac{x_1 + kx_2}{1+k}, \frac{y_1 + ky_2}{1+k}, \frac{z_1 + kz_2}{1+k} \right)$$

Angle  $\phi$  between two lines with direction cosines  $l, m, n$  :

$$\cos \phi = \frac{\pm(l'l' + mm' + nn')}{\sqrt{(l^2 + m^2 + n^2)(l'^2 + m'^2 + n'^2)}}$$

Distance from  $P_1(x_1, y_1, z_1)$  to plane  $Ax + By + Cz + D = 0$  is

$$\left| \frac{Ax_1 + By_1 + Cz_1 + D}{\sqrt{A^2 + B^2 + C^2}} \right|$$

Plane distance  $p$  from origin, direction cosines of normal  $l, m, n$ ,

$$lx + my + nz = p.$$

Line through  $(x_1, y_1, z_1)$ , direction cosines  $l, m, n$ ,

$$\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n} = t.$$

# CALCULUS

## I. INFINITE SERIES

### Taylor's Theorem

$$f(a+x) = f(a) + xf'(a) + \frac{x^2}{2!}f''(a) + \dots + \frac{x^{r-1}}{(r-1)!}f^{(r-1)}(a) + \dots,$$

with 'remainder term'  $\frac{x^r}{r!}f^{(r)}(a + \theta x)$ , where  $0 < \theta < 1$ .

### Maclaurin's Theorem

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \dots + \frac{x^{r-1}}{(r-1)!}f^{(r-1)}(0) + \dots,$$

with 'remainder term'  $\frac{x^r}{r!}f^{(r)}(\theta x)$ , where  $0 < \theta < 1$ .

$$\exp x \equiv e^x = 1 + x + \frac{x^2}{2!} + \dots + \frac{x^r}{r!} + \dots \quad *$$

$$\log_e(1+x) \equiv \ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots + (-1)^{r-1} \frac{x^r}{r} + \dots$$

valid for  $-1 < x \leq 1$ .

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots + (-1)^r \frac{x^{2r+1}}{(2r+1)!} + \dots \quad *$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots + (-1)^r \frac{x^{2r}}{(2r)!} + \dots \quad *$$

$$\sinh x = \frac{1}{2}(e^x - e^{-x}) = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots + \frac{x^{2r+1}}{(2r+1)!} + \dots \quad *$$

$$\cosh x = \frac{1}{2}(e^x + e^{-x}) = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots + \frac{x^{2r}}{(2r)!} + \dots \quad *$$

\* These series are valid for all finite  $x$ .

Probability generating function  $G(z)$

$$G(z) = P(0) + P(1)z + P(2)z^2 + \dots + P(r)z^r + \dots,$$

where  $P(r) = P(X = r)$

Binomial distribution  $(X, p, N)$

$$P(X = k) = \binom{N}{k} p^k (1-p)^{N-k}$$

$$E(X) = Np$$

$$\text{Var}(X) = Np(1-p)$$

$$G(z) = [pz + (1-p)]^N$$

Poisson distribution  $(X, m)$

$$P(X = k) = \frac{e^{-m} m^k}{k!}$$

$$E(X) = m$$

$$\text{Var}(X) = m$$

$$G(z) = e^{-m} e^{mz}$$

Normal distribution

If  $X$  is distributed  $N(\mu, \sigma^2)$  then  $\frac{X - \mu}{\sigma}$  is distributed  $N(0, 1)$

where  $\sigma$  is the standard deviation and  $\sigma^2$  is the variance.

II DERIVATIVES

$f(x)$	$f'(x)$
$x^n$	$nx^{n-1}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$e^x$	$e^x$
$a^x (a > 0)$	$a^x \ln a$
$\log_e x \equiv \ln x$	$\frac{1}{x}$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$uv$	$u'v + uv'$
$\frac{v}{u}$	$\frac{u'v - uv'}{u^2}$

PROBABILITY

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A) \times P(B | A)$$

Probability laws

Discrete variable X with probability function $P(X = x)$	Continuous variable X with probability density function $f(X)$
$F(x_0) = P(X \leq x_0)$ $= \sum_{x \leq x_0} P(x)$	$F(x) = P(X > x)$ $= \int_{x_0}^{-\infty} f(x) dx$
Expectation of X $E(X) = \sum xP(X = x)$	$E(X) = \int xf(x) dx$
Expectation of $g(x)$ $E[g(x)] = \sum g(x)P(X = x)$	$E[g(X)] = \int g(x)f(x) dx$
Variance $\sigma^2$ $\operatorname{Var}(X) = E\{X^2\} - E(X)^2$	
Covariance $\operatorname{Cov}(X_1, X_2) = E\{X_1 - E(X_1)\}\{X_2 - E(X_2)\}$	
Correlation coefficient $\rho_{12}(X_1, X_2)$ $\rho_{12} = \frac{\operatorname{Cov}(X_1, X_2)}{\sqrt{\operatorname{Var}(X_1)\operatorname{Var}(X_2)}}$	
Linear regression coefficient, $\beta_{12}$ , for $X_1$ on $X_2$ $\beta_{12} = \frac{\operatorname{Cov}(X_1, X_2)}{\operatorname{Var}(X_2)}$	

III INTEGRALS (Constants of integration are omitted;  $\ln a \equiv \log_e a$ )

$f(x)$	$\int f(x)dx$
$\frac{1}{\sqrt{(a^2 - x^2)}}$	$\sin^{-1} \left( \frac{x}{a} \right)$
$\frac{1}{(a^2 + x^2)}$	$\frac{1}{a} \tan^{-1} \left( \frac{x}{a} \right)$
$\frac{1}{\sqrt{(a^2 + x^2)}}$	$\ln\{x + \sqrt{(x^2 + a^2)}\}$ or $\sinh^{-1} \left( \frac{x}{a} \right)$
$\frac{x}{\sqrt{(a^2 + x^2)}}$	$\sqrt{(a^2 + x^2)}$
$\frac{1}{\sqrt{(a^2 - x^2)}}$	$\ln\{x + \sqrt{(x^2 - a^2)}\}$ or $\cosh^{-1} \left( \frac{x}{a} \right)$
$\sin x$	$-\cos x$
$\cos x$	$\sin x$
$\tan x$	$\ln(\sec x)$
$\cot x$	$\ln(\sin x)$
$\sec x$	$\ln(\sec x + \tan x)$ or $\ln \left\{ \tan \left( \frac{x}{2} + \frac{\pi}{4} \right) \right\}$
$\operatorname{cosec} x$	$\ln \tan \frac{x}{2}$
$\cosh x$	$\sinh x$
$\sinh x$	$\cosh x$
$u \frac{dv}{dx}$	$uv - \int v \frac{du}{dx} dx$

MECHANICS

Centre of mass

Arc, radius $r$ , angle $2\theta$	$r \sin \theta / \theta$ from centre
Sector of circle, radius $r$ , angle $2\theta$	$\frac{2}{3} r \sin \theta / \theta$ from centre
Hemisphere, radius $r$	$\frac{3}{8} r$ from centre
Hemispherical shell, radius $r$	$\frac{1}{2} r$ from centre
Solid cone, height $h$	$\frac{1}{2} h$ from vertex
Conical shell, height $h$	$\frac{2}{3} h$ from vertex

Moments of inertia

Rod, length $2l$ , about perpendicular axis through centre	$\frac{1}{3} ml^2$
Disc, radius $r$ , about perpendicular axis through centre	$\frac{1}{2} mr^2$
Hoop, radius $r$ , about diameter	$\frac{1}{2} mr^2$
Solid sphere, radius $r$ , about diameter	$\frac{2}{5} mr^2$
Spherical shell, radius $r$ , about a diameter	$\frac{2}{3} mr^2$
Parallel axes theorem	$I_A = I_G + M(GA)^2$
Perpendicular axes theorem for a lamina	$I_{oz} = I_{ox} + I_{oy}$

Simple harmonic motion

$$\frac{d^2y}{dt^2} = -\omega^2 x, \left( \frac{dx}{dt} \right)^2 = \omega^2 (a^2 - x^2), x = a \sin(\omega t + \epsilon)$$

Compound pendulum

$$\text{Period} = 2\pi \sqrt{(k^2 + h^2)/gh}$$

Components of acceleration

$\ddot{r} - r\dot{\theta}^2$  along radius vector

$2\dot{r}\dot{\theta} + r\ddot{\theta}$  perpendicular to radius vector

V APPROXIMATIONS

Trapezoidal Rule:

$$\int_b^a y \, dx \approx \frac{1}{2}h \{ (y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1}) \}$$

Simpson's rule (n even)

$$\int_b^a y \, dx \approx \frac{1}{3}h \{ (y_0 + y_n) + 4(y_1 + y_3 + \dots + y_{n-1}) + 2(y_2 + y_4 + \dots + y_{n-2}) \}$$

Newton's approximation to a root of  $f(x) = 0$ :

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

VECTORS

Line through point, position vector  $\mathbf{a}$ , parallel to  $\mathbf{b}$

$$r = \mathbf{a} + t\mathbf{b}$$

Position vector of a point dividing the line joining P, Q with position

$$\text{vectors } \mathbf{p}, \mathbf{q} \text{ in the ratio } \lambda : \mu \text{ is } \frac{\lambda\mathbf{p} + \mu\mathbf{q}}{\lambda + \mu}$$

Plane through point, position vector  $\mathbf{a}$ , perpendicular to  $\mathbf{n}$

$$(\mathbf{r} - \mathbf{a}) \cdot \mathbf{n} = 0$$

Scalar product =  $\mathbf{a}_1 \cdot \mathbf{a}_2 = a_1 a_2 \cos \theta = x_1 x_2 + y_1 y_2 + z_1 z_2$

$$\text{Vector product} = \mathbf{a}_1 \times \mathbf{a}_2 = a_1 \wedge a_2 = a_1 a_2 \sin \theta \mathbf{h} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix}$$

IV APPLICATIONS

For a curve  $y = f(x)$ ,  $a \leq x \leq b$ .

$$\text{Area between curve and x-axis} = \int_b^a y \, dx$$

$$\text{Mean value} = \frac{1}{b-a} \int_b^a y \, dx$$

$$\text{Volume of revolution about x-axis} = \pi \int_b^a y^2 \, dx$$

Centroid of area between curve and x-axis has coordinates

$$\bar{x} = \frac{\int_b^a xy \, dx}{\int_b^a y \, dx}; \quad \bar{y} = \frac{\int_b^a \frac{1}{2}y^2 \, dx}{\int_b^a y \, dx}$$

Centroid of solid of revolution about x-axis:

$$\bar{x} = \frac{\int_b^a xy^2 \, dx}{\int_b^a y^2 \, dx}$$

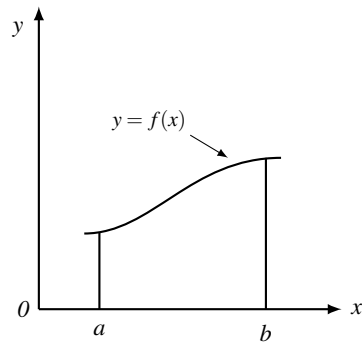
**For the area shown in Figure 1**

$$\text{First moment about } x\text{-axis} = \int_a^b \frac{1}{2}y^2 dx$$

$$\text{First moment about } y\text{-axis} = \int_a^b xy dx$$

$$\text{Second moment about } x\text{-axis} = \int_a^b \frac{1}{3}y^3 dx$$

$$\text{Second moment about } y\text{-axis} = \int_a^b x^2y dx$$



**Fig. 1**

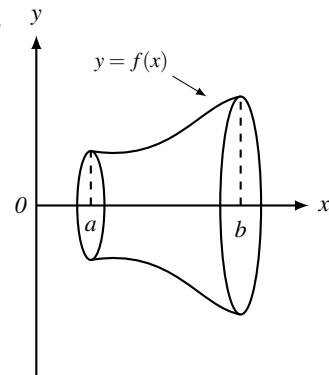
**For the solid of revolution shown in Figure 2**

$$\text{First moment about } xy\text{-plane} = 0$$

$$\text{First moment about } yz\text{-plane} = \pi \int_a^b xy^2 dx$$

$$\text{Second moment about } x\text{-axis} = \pi \int_a^b \frac{1}{2}y^4 dx$$

$$\text{Second moment about } y\text{-axis} = \pi \int_a^b y^2 \left( x^2 + \frac{y^2}{4} \right) dx$$



**Fig. 2**

$$\text{Length of arc} = \int_a^b \sqrt{\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}} dx = \int_{t_1}^{t_2} \sqrt{(x^2 + y^2)} dt$$

$$\begin{aligned} \text{Area of surface of revolution} &= 2\pi \int_a^b y \sqrt{\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}} dx \\ &= 2\pi \int_{t_1}^{t_2} y \sqrt{(x^2 + y^2)} dt \end{aligned}$$

$$\text{Radius of curvature } \rho = \frac{\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{3/2}}{\frac{d^2y}{dx^2}} = \frac{(x^2 + y^2)^{3/2}}{x\ddot{y} - \dot{x}\dot{y}}$$

**Polar Coordinates**

$$\text{Area enclosed by curve} = \frac{1}{2} \int_{\theta_1}^{\theta_2} r^2 d\theta$$

$$\text{Length of arc} = \int_{\theta_1}^{\theta_2} \sqrt{\left\{r^2 + \left(\frac{dr}{d\theta}\right)^2\right\}} d\theta = \int_{r_1}^{r_2} \sqrt{\left\{1 + r^2 \left(\frac{d\theta}{dr}\right)^2\right\}} dr$$

$$\text{Radius of curvature } \rho = r \left/ \frac{dp}{dr} \right.$$