Mock Assessment Test

Mathematics Tutorials*

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Instructions

The goal of this test is to prepare you for your initial mathematical assessment in partial fulfilment for the award of B.Sc. Construction Engineering (Hons) at MCAST (ETMTH-506-1514).

Read the following instructions carefully.

- This test consists of **5 questions** and carries **33 marks**.
- You have **three hours** to complete this test.
- Attempt **all** questions.

Hints and solutions are provided at the back of the test paper.

*https://maths.com.mt

1. Consider the real-valued function f defined by

$$f(x) = 12x^3 - 16x^2 - 5x + 3.$$

   (a) Show that $f(\frac{1}{3}) = 0$.

   (b) Using the factor theorem or otherwise, express the function f fully factorised.

   (c) Hence or otherwise, sketch the graph defined by $y = f(x)$, clearly labelling any x- and y-intercepts.

   (d) Now consider the real-valued function g given by

$$g(x) = 2x - 3.$$

   On the same pair of axes drawn for part (c), sketch the graph of the straight line $y = 6 \times g(x)$, finding and labelling clearly any intersection points of the line with the cubic f.

[1, 2, 2, 2 marks]

2. Santa's elves are building toys for boys and girls in their workshop before Christmas day. Toy A requires 5 blocks of wood and 12 screws, whereas toy B requires 10 blocks of wood and 9 screws. The workshop is currently fully stocked with supplies. Moreover, a delivery of 250 000 blocks of wood and 360 000 screws is coming soon, so at least this many blocks of wood and screws have to be used to make room for the new supplies before the delivery arrives.

   When completed, each toy of type A weighs 200 g, whereas each toy of type B weighs 250 g. The combined load of both toys must go in Santa's sleigh, so the lesser the total weight, the better.

   Given that the elves can produce at most 30 000 toys of each type before the delivery arrives, use linear programming to minimise the combined weight.

[7 marks]

3. The total number of sales $s(t)$ of a book after t days of publishing is modelled by the function

$$s(t) = \lambda(1 - e^{-\mu t}),$$

where λ, μ are fixed constants. After the first day, the book sold 120 copies, and on the next day, the book sold an additional 60 copies.

(a) Determine the constants λ and μ.
[Note: Give μ in the form $\ln k$ where k is a whole number.]

(b) According to the model, how many copies are sold on the third and fourth days?

[4, 1 marks]

4. Consider the following system of equations.

$$\begin{cases} 5x - 2y = 16 \\ 7x + 3y = 5 \end{cases}$$

Solve this system for x and y by inverting the matrix of coefficients.

[7 marks]

5. A building 36 m high casts a shadow 90 m long.

(a) Express the length s of the shadow of a building in terms of its height b.

(b) How high does a building have to be to cast a shadow 105 m long?

A fort manned by 45 men has food and water for 100 days.

(c) How long would the same supplies last if the fort were manned by 50 men?

(d) After 25 days, 15 out of the 45 men leave the fort. How many days can the remaining supplies provide for the remaining men in the fort?

[1, 1, 2, 3 marks]

End of Paper

Hints and Solutions

Don't look at these till after you attempt the whole test!

1. (a) *Hint: Just substitute* $x = \frac{1}{3}$ *in* f *everywhere to show that* $f(\frac{1}{3}) = 0$.

   (b) *Hint: Part (a) already gave us that* $f(\frac{1}{3}) = 0$, *which by the factor theorem gives us that* $(3x - 1) \mid f$, *so determine* $\frac{f}{3x-1}$ *by long division to find the remaining quadratic factor, and factorise that if possible.*

   Answer: $f(x) = (3x - 1)(2x + 1)(2x - 3)$.

   (c) The sketch is given in figure 1.

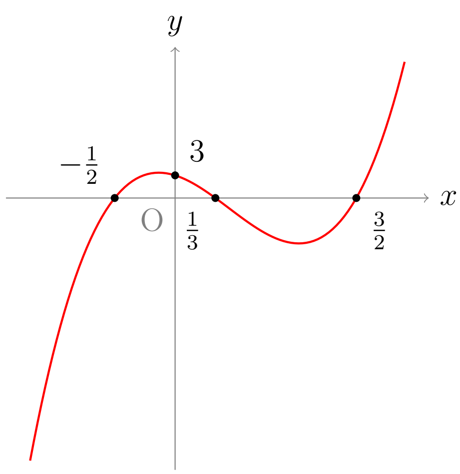

FIGURE 1: $y = f(x)$

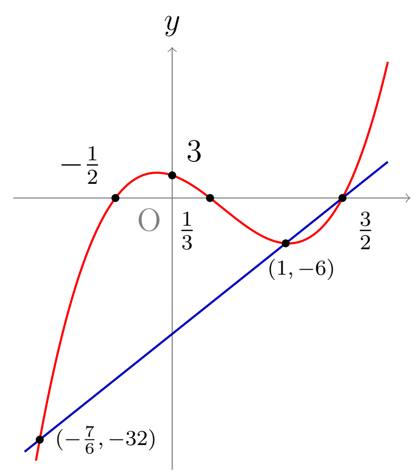

FIGURE 2: $y = f(x)$ and $y = 6g(x)$ on same axes

   (d) *Hint: To find where* $y = 6 \times g(x)$ *intersects* $y = f(x)$, *solve the equation* $f(x) = 6g(x)$, *that is,* $(3x-1)(2x+1)(2x-3) = 6(2x-3)$. *This gives* $x = -\frac{7}{6}$, $x = 1$ *and* $x = \frac{3}{2}$, *whose corresponding points are* $(-\frac{7}{6}, -32)$, $(1, -6)$ *and* $(\frac{3}{2}, 0)$.

   The sketch is given in figure 2.

2. *Hint: let* a *be the number of toys of type* A, *and* b *the number of toys of type* B. *Then the feasible region* $\mathcal{R}$ *in the plane is the quadrilateral*

defined by the given constraints

$$\begin{cases} 5a + 10b \geqslant 250\,000 \\ 12a + 9b \geqslant 360\,000 \\ a \leqslant 30\,000 \\ b \leqslant 30\,000, \end{cases} \tag{1}$$

and we wish to determine the value of

$$\min_{(a,b)\in\mathcal{R}} (200a + 250b).$$

It suffices to evaluate the objective function $200a + 250b$ *at the vertices of the quadrilateral* $\mathcal{R}$*, which are given when considering the cases of equality in* (1)*:* $(30\,000, 30\,000)$*,* $(7\,500, 30\,000)$*,* $(30\,000, 10\,000)$*, and* $(18\,000, 16\,000)$*. It is then only a matter of evaluation to conclude that* $200a + 250b$ *is minimal when* $a = 18\,000$ *and* $b = 16\,000$*.*

Answer: Minimum weight is attained when 18 000 toys of type A and 16 000 toys of type B are produced.

3. (a) *Hint: Since after the first day,* 120 *copies are sold, we have* $s(1) = 120$*. Since on the second day, an additional* 60 *copies are sold, then the total on the second day is* $120 + 60 = 180$*, so we also have that* $s(2) = 180$*. Thus we have the simultaneous equations*

$$\begin{cases} \lambda(1 - e^{-\mu}) = 120 & (2) \\ \lambda(1 - e^{-2\mu}) = 180. & (3) \end{cases}$$

One possible strategy to solve this system: from (2) *we have that* $\lambda = \frac{120}{1-e^{-\mu}}$*, which we can then substitute into* (3)*, and then the equation simplifies to* $2(e^{-\mu})^2 - 3(e^{-\mu}) + 1 = 0$*. This can be solved as a quadratic to yield the possibilities* $e^{-\mu} = \frac{1}{2}$*,* $e^{-\mu} = 1$*. The first gives* $\mu = \ln 2$*, the second gives* $\mu = 0$*, which is not meaningful. Thus we have* $\mu = \ln 2$*, and then substituting in the equation for* λ *gives* $\lambda = 240$*.*

Answer: $\mu = \ln 2$, $\lambda = 240$.

(b) *Hint: On the third day,* $s(3) = 210$*, but this is the total including the first two days (*$s(2)$*), so on the third day alone,* $s(3) - s(2) =$

$210 - 180 = 30$ *copies are sold. Similarly, on the fourth day,* $s(4) - s(3) = 225 - 210 = 15$ *copies are sold.*

Answer: 30 on the third day, 15 on the fourth day.

4. *Hint: The system is equivalent to the matrix equation* $\mathbf{Ax} = \mathbf{b}$*, where*

$$\mathbf{A} = \begin{pmatrix} 5 & -2 \\ 7 & 3 \end{pmatrix}, \qquad \mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix} \qquad \textit{and} \qquad \mathbf{b} = \begin{pmatrix} 16 \\ 5 \end{pmatrix}.$$

Thus the solution vector $\mathbf{x}$ *is given by* $\mathbf{x} = \mathbf{A}^{-1}\mathbf{b}$*, which simplifies to* $\mathbf{x} = (2, -3)$*.*

Answer: $x = 2$, $y = -3$.

5. (a) *Hint: The length of a shadow grows proportionally with the height of its building, so* $s \propto b$*, or equivalently,* $s = kb$ *for some fixed proportionality constant* k*. In particular, this is true when* $s = 90$ *and* $b = 36$*, so we get* $90 = 36k$*, which gives* $k = \frac{5}{2}$*.*

   Answer: $s = \frac{5}{2}b$.

   (b) *By the previous part, we simply put* $s = 105$ *and solve for* b*.*

   Answer: 42 m.

   (c) *Hint: If the number of men grows, the number of days for which the supplies last decreases, so they are inversely proportional. If* m *is the number of men, and* d *is the number of days, we have* $m \propto \frac{1}{d}$*, or equivalently,* $m = \frac{k}{d}$ *for some fixed* k*. Since we have the instance* $m = 45$ *and* $d = 100$*, we can solve similarly to part (a) for the constant* k *to get* $k = 4500$*. Thus* $m = \frac{4500}{d}$*. Now putting* $m = 50$ *and solving for* d *yields the desired result.*

   Answer: 90 days.

   (d) *Hint: After 25 days, there are 75 days' worth of food left for 45 men. So at this point in time, the relation changes: we still have* $m \propto \frac{1}{d}$*, but the instance changes to* $m = 45$ *and* $d = 75$*, which gives a proportionality constant of* $k = 3375$*. So now* $m = \frac{3375}{d}$*. But since* m *has now decreased by 15, the 75 days' worth of food is actually* $\frac{3375}{45-15} = 112\frac{1}{2}$ *days' worth of food for 30 men.*

   Answer: $112\frac{1}{2}$ days.